

11. (15 points) Consider the non-homogeneous equation: $y''' + y'' - 8y' - 12y = 12x + 8$

a. What is form of the general solution of a differential equation?

(1)

$$y = y_{\text{homo.}} + y_{\text{part.}}$$

b. Find the general solution of the associated homogenous equation.

(5)

$$y''' + y'' - 8y' - 12y = 0$$

$$\lambda^3 + \lambda^2 - 8\lambda - 12 = 0$$

$$(\lambda - 3)(\lambda + 2)^2 = 0$$

$$\lambda = 3, -2$$

$$y_h = c_1 e^{3x} + c_2 e^{-2x} + c_3 x e^{-2x}$$

c. Find some particular solution of the non-homogeneous equation.

(4)

$$y_p = -x$$

$$\text{check: } y''' + y'' - 8y' - 12y = 0 + 0 + 8 + 12x \checkmark$$

(1)

d. Using the above information, what is the general solution of the non-homogeneous differential equation?

$$y = -x + c_1 e^{3x} + c_2 e^{-2x} + c_3 x e^{-2x}$$

e. Find the particular solution of the non-homogeneous differential equation which satisfies the initial conditions $y(0) = 0$, $y'(0) = 2$, $y''(0) = 13$

(4)

$$0 = c_1 + c_2$$

$$y' = -1 + 3c_1 e^{3x} + (-2c_2 + c_3 - 2c_3 x) e^{-2x}$$

$$2 = -1 + 3c_1 - 2c_2 + c_3 \rightarrow 3 = 3c_1 - 2c_2 + c_3$$

$$y'' = 9c_1 e^{3x} + (-2c_3 e^{-2x} + (4c_2 - 2c_3 + 4c_3 x) e^{-2x})$$

$$13 = 9c_1 - 4c_3 + 4c_2$$

$$\Rightarrow c_1 = 1, c_2 = -1, c_3 = -2$$

$$y = -x + e^{3x} - e^{-2x} - 2x e^{-2x}$$